WOTCA 2018

Workshop on Operator Theory,
Complex Analysis, and Applications

BOOK OF ABSTRACTS

Department of Mathematics
University of Minho, Guimarães, Portugal
June 25-28, 2018

The Dirichlet space on the bidisc

Nicolla Arcozzi

Universit di Bologna, Italy

E-mail address: *nicola.arcozzi@unibo.it*

Abstract

The Dirichlet space on the bidisc might be seen as the tensor product of two copies of the Dirichlet space on the disc. Its theory, however, has its own specific features, which boil down to the development of a bi-parameter potential theory, in which the Maximum Principle does not hold, but the Stron Capacitary Inequality does. We discuss here Carleson measures and the characterization of the multiplier algebra.

Work in collaboration with Pavel Mozolyako, Karl-Mikael Perfekt, Giulia Sarfatti.

Generalized kq-representation and bi-coherent states

Fabio Bagarello

University of Palermo
INFN, Napoli

E-mail address: *fabio.bagarello@unipa.it*

Abstract

We introduce a generalized kq-representation, related to a suitable non self-adjoint version of the momentum and position operators. Certain linear combinations of these define ladder operators already considered in connection with *pseudo-bosons*, and *bi-coherent states* can be constructed out of them. We show how this kq-representation can be used to prove completeness of a discrete set of these bi-coherent states in $\mathcal{L}^2(\mathbf{R})$.

Keywords: coherent states, kq-representation, position and momentum operators.

Real spectra in non self-adjoint operators and extensions of quantum mechanics

Natália Bebiano[1,2] **and João da Providência**[3,4]

[1]*Department of Mathematics, University of Coimbra, Portugal*
[2]*CMUC - Centro de Matemática da Universidade de Coimbra*
[3]*CFisUC, University of Coimbra*
[4]*Departamento de Física, 3001-454 Coimbra, Portugal*

E-mail address: *bebiano@mat.uc.pt; providencia@teor.fis.uc.pt*

Abstract

The formulation of conventional quantum mechanics is based on the theory of self-adjoint operators which describe *observables* and whose eigenvalues are the possible results of the respective measurements. In particular, the Hamiltonian operator is self-adjoint, has a real set of eigenvalues and corresponding orthonormal eigenfunctions. Certain relativistic extensions of quantum mechanics lead to non self-adjoint Hamiltonian operators with real spectra, which motivated an intense research activity, namely on the so called *PT-quantum mechanics*. (Here P and T are, respectively, the *parity* and the *time reversal* operators.)

There have been attempts to develop the so called *quasi-Hermitian quantum mechanics* where the Hamiltonian H is represented by a *quasi-self-adjoint* operator, that is, H satisfies the quasi-self-adjointness relation

$$H^*\Theta = \Theta H, \tag{1}$$

whit $\Theta = T^*T$ a positive, bounded and boundedly invertible operator, called a *metric*. The concept of quasi-self-adjointness goes back to Dieudonné and is of remarkable interest in the context of quantum mechanics with non-self-adjoint Hamiltonians. A consistent quantum theory can be built for these operators, with the metric defining a new inner product in the underlying Hilbert space, relatively to which H becomes self-adjoint via the similarity transformation THT^{-1}. However, it is in general difficult to decide quasi-self-adjointness of an operator. A necessary condition is the reality of the spectrum, $\sigma(H)$. If T is bounded and boundedly invertible, then the spectra of THT^{-1} and H coincide and the eigenfunctions share basis properties. It is not very common to find in the literature non-self-adjoint models for which a metric is constructed, neither the existence of a metric operator is guaranteed. Problems arise if T or T^{-1} are unbounded, such as it may happen that the

spectrum of H is purely discrete, while THT^{-1} has no eigenvalues. Further, unbounded transformations may turn an orthogonal eigenbasis into a set of functions that do not form any kind of basis.

The talk is outlined as follows. A non-self-adjoint harmonic oscillator is considered and its basic properties are stated. Complete systems of eigenfunctions of H and of its adjoint H^* are computed. The quasi-Hermiticity of the Hamiltonian is investigated. Due to the non-self-adjointness of H, its eigenfunctions are not expected to form a basis of the underlying Hilbert space. This issue is discussed in relation to our model. The existence of a metric operator which renders the Hamiltonian quasi-Hermitian, is analyzed. The physical Hilbert space, which is required for the physical interpretation of the model, is introduced, by defining a convenient inner product.

Keywords: bosonic operators, non-Hermitian Hamiltonans, spectral analysis, metric.

Examples of Generalized Bi-Circular Idempotents on Spaces of Continuously Differentiable Functions

Fernanda Botelho[1] **and Takeshi Miura**[2]

[1]*Department of Mathematical Sciences, The University of Memphis Memphis, TN 38152, USA*
[2]*Department of Mathematics, Niigata University, Japan*

E-mail address: *mbotelho@memphis.edu; miura@math.sc.niigata-u.ac.jp*

Abstract

The Mazur-Ulam theorem formulates that some translation of a surjective and distance preserving map on a complex normed space is real linear. In this talk we present results concerning a class of idempotent maps, defined on normed spaces of complex valued differentiable functions, associated with a surjective isometry. More precisely, we study idempotents with the property that the complex hull defined by one such idempotent and its complementary must contain a surjective isometry. The definition introduced here follows closely the definition of generalized bi-circular projection, however the maps involved are not assumed to be linear.

Keywords: generalized bi-circular idempotents, the Mazur-Ulam theorem.

Perturbations of Gibbs semigroups and non-selfadjoint Schrödinger operators

Lyonell Boulton

Department of Mathematics and Maxwell Institute for Mathematical Sciences, Heriot-Watt University, Edinburgh, United Kingdom

E-mail address: *l.boulton@hw.ac.uk*

Abstract

Let $-T$ be the generator of a C_0 one-parameter semigroup e^{-Tt} which is of finite trace for all $t > 0$ (a Gibbs semigroup). Let A be another closed operator, T-bounded with T-bound equal to zero. In general $-(T+A)$ might or might not be the generator of a Gibbs semigroup. In the first half of this talk we give sufficient conditions on A so that $-(T+A)$ is the generator of a Gibbs semigroup. We determine these conditions in terms of the convergence of the Dyson-Phillips expansion corresponding to the perturbed semigroup in suitable Schatten-von Neumann norms. They are optimal in a sense to be made precise during the talk.

In the second half of the talk we consider $T = H_\vartheta$ and $A = V$ where H_ϑ is the non-selfadjoint harmonic oscillator

$$H_\vartheta = -\mathrm{e}^{-\mathrm{i}\vartheta}\partial_x^2 + \mathrm{e}^{\mathrm{i}\vartheta}x^2$$

acting on $L^2(\mathbb{R})$, $0 < \vartheta < \frac{\pi}{2}$ and $V : \mathbb{R} \longrightarrow \mathbb{C}$ is a locally integrable potential such that

$$|V(x)| \leq a|x|^\alpha + b \qquad\qquad \forall x \in \mathbb{R}$$

for some $0 \leq \alpha < 2$, $a > 0$ and $b \in \mathbb{R}$. We show that $-(H_\vartheta + V)$ is the generator of a Gibbs semigroup $\mathrm{e}^{-(H_\vartheta+V)\tau}$ for $|\arg\tau| \leq \frac{\pi}{2} - |\vartheta|$. By virtue of a non-selfadjoint version of an inequality due to Ginibre and Gruber found in 2001 by Cachia and Zagrebnov, it is possible to deduce this on an open sector. But what is remarkable and far from obvious here is the fact that the finite trace property extends all the way to the edges of the sector above. To achieve the latter, we invoke the first part of the talk and show that the Dyson-Phillips expansion converges in this case in r Schatten-von Neumann norm for $r > \frac{4}{2-\alpha}$.

Keywords: perturbation of Gibbs semigroups, Dyson-Phillips expansion, non-selfadjoint Schrödinger operators.

Weyl space-times and Riemann-Hilbert factorization

Gabriel Lopes Cardoso

Instituto Superior Técnico
CAMGSD, Department of Mathematics, Portugal

E-mail address: *gcardoso@math.ist.utl.pt*

Abstract

We describe a few recent developments in the context of the Riemann-Hilbert approach to Einstein's field equations.

Based on joint work with Cristina Câmara, Thomas Mohaupt, Suresh Nampuri and João Serra.

Keywords: Riemann-Hilbert factorization, Weyl space-times, Einstein's field equations.

Convolutions for quadratic-phase Fourier integral operators and some of their applications

L. P. Castro[1,2], **L. T. Minh**[3] **and N. M. Tuan**[4]

[1] *University of Aveiro*
[2] *CIDMA–Center for Research and Development in Mathematics and Applications*
[3] *Hanoi Architectural University*
[4] *Vietnam National University*

E-mail address: *castro@ua.pt*

Abstract

We will present new convolutions for quadratic-phase Fourier integral operators (which include, as subcases, e.g., the fractional Fourier transform and the linear canonical transform). The structure of these convolutions is based on properties of the mentioned integral operators and takes profit of weight-functions associated with some amplitude and Gaussian functions. Therefore, the fundamental properties of that quadratic-phase Fourier integral operators are also studied (including a Riemann-Lebesgue type lemma, invertibility results, a Plancherel type theorem and a Parseval type identity). As applications, we obtain new Young type inequalities, the asymptotic behaviour of some oscillatory integrals, and the solvability of convolution integral equations.

The talk is based on the work [L.P. Castro, L.T. Minh, N.M. Tuan, Mediterr. J. Math. (2018) 15:13 https://doi.org/10.1007/s00009-017-1063-y].

Keywords: Convolution, Young inequality, oscillatory integral, convolution integral equation, fractional Fourier transform, linear canonical transform.

The principle of locally made simpler but harder

Man - Duen Choi

University of Toronto, Canada

E-mail address: *choi@math.toronto.edu*

Abstract

In physics, the principle of locality says that each phenomenon is influenced directly by the local surroundings. This could be translated to a simple mathematical statement of no wisdom at all. Nevertheless, with extravagant description of the obvious truth, or fascinating explanation of absurdity, the principle may become a big law/theory/theorem or a paradox to shake your body/mind. Here, I will give an expository talk in the settings of direct sums and tensor products as related to quantum information theory.

A differential geometry in the set of contractions

E. Andruchow, Gustavo Corach, L. Recht

Fac. de Ingeniería-Universidad de Buenos Aires
Instituto Argentino de Matemática Alberto P. Calderón- CONICET,
Argentina

E-mail address: *gcorach@fi.uba.ar*

Abstract

We study the tangent bundle of the manifold of all positive invertible operators on a Hilbert space and show that it can be identified with an hyperbolic geometry on the set of all contractions on that Hilbert space. This is an operator version of the classical Poincaré disk model of the hyperbolic plane.

Simultaneous bounded extension of two operators

Marko Djikić

Department of Mathematics, Faculty of Sciences and Mathematics,
University of Niš, Serbia

E-mail address: *marko.djikic@gmail.com*

Abstract

We will discuss the following problem: when do two bounded operators, defined on different subspaces of the same Hilbert space, have a simultaneous bounded (or closed) extension to the whole space? While discussing such a problem we will recall some old results of Izumino on quotient operators, but we will also invoke new results regarding unbounded pseudoinverses and orthogonal projections.

Keywords: coherent operators, quotient operators, Moore-Penrose inverse

Eigenvalues of compactly perturbed operators via entropy numbers

Marcel Hansmann

Chemnitz University of Technology, Germany

E-mail address: *marcel.hansmann@mathematik.tu-chemnitz.de*

Abstract

If we add a compact operator K to some free operator A, the essential spectra of $A + K$ and A will coincide. However, the perturbation can create (infinitely many) new discrete eigenvalues, which can accumulate at the joint essential spectrum only. In recent years, there has been a considerable interest in quantitative results on the rate of this accumulation, given some more restricted assumptions on K. Here the focus of attention has been non-selfadjoint operators in Hilbert spaces and perturbations K from the Schatten-von Neumann ideals $\mathcal{S}_p$ (i.e., the singular values of K are in l_p). In this case, results on the discrete eigenvalues of $A+K$ where proven by (essentially) two different methods: (i) Complex analysis and infinite determinants (i.e., by identifying the eigenvalues with the zeroes of a suitable perturbation determinant), or (ii) Algebraic methods relying on special properties of the Schatten-von Neumann ideals.

While it is fair to say that the Hilbert space case is by now quite well understood, the same cannot be said for operators on general Banach spaces. In particular, until now the only available method of proof in this more general setting is the use of complex analysis and infinite determinants, mimicking the ideas used in the Hilbert space context.

It is the aim of this talk to describe a new and determinant-free method of proof for operators in general Banach spaces. The main tool in this new method is a classical estimate by B. Carl between the eigenvalues and entropy numbers of a compact operator. If time permits, we will sketch some applications of our new results in the study of Schrödinger operators on the hyperbolic plane, where the L_p-spectrum is p-dependent.

Non-self-adjoint graphs

A. Hussein[1], **D. Krejčiřík**[2], **P. Siegl**[3]

[1] *TU Darmstadt*
Department of Mathematics
[2] *Czech Technical University in Prague*
Department of Mathematics
[3] *Queen's University Belfast*
Mathematical Sciences Research Centre

E-mail address: *hussein@mathematik.tu-darmstadt.de*

Abstract

On finite metric graphs Laplace operators subject to various classes of non-self-adjoint boundary conditions imposed at graph vertices are considered. Spectral properties are investigated, in particular similarity transforms to self-adjoint operators. Concrete examples are discussed exhibiting that non-self-adjoint boundary conditions can yield to unexpected spectral features.

Keywords: non-self-adjoint operators, operators on metric graphs.

Complexifications of real Banach spaces and their isometries

D. Ilišević[1], **B. Kuzma**[2,3], **C.-K. Li**[4], **and E. Poon**[5]

[1] *University of Zagreb, Croatia*
[2] *Department of Mathematics, University of Primorska, Slovenia*
[3] *Institute of Mathematics, Physics, and Mechanics, Slovenia*
[4] *Department of Mathematics, College of William and Mary, Williamsburg, VA 23187-8795, USA*
[5] *Department of Mathematics, Embry-Riddle Aeronautical University, Prescott AZ 86301, USA*

E-mail address: *ilisevic@math.hr*

Abstract

Every norm $\|\cdot\|$ on a real Banach space $\mathcal{X}$ induces a norm on the complex linear space $\mathcal{X}_{\mathbb{C}} = \mathcal{X} + i\mathcal{X} = \{x + iy : x, y \in \mathcal{X}\}$ by

$$\|x + iy\|_{\mathfrak{T}} = \sup\{\|x\cos\theta + y\sin\theta\| : \theta \in [0, 2\pi]\}.$$

This norm is known as the Taylor complexification norm. If A is an isometry for the real Banach space $(\mathcal{X}, \|\cdot\|)$, then its complexification (sending $x + iy$ to $Ax + iAy$), which is for brevity again denoted by A, satisfies

$$\begin{aligned}\|A(x+iy)\|_{\mathfrak{T}} &= \sup_{\theta\in[0,2\pi]} \|Ax\cos\theta + Ay\sin\theta\| = \sup_{\theta\in[0,2\pi]} \|A(x\cos\theta + y\sin\theta)\| \\ &= \sup_{\theta\in[0,2\pi]} \|x\cos\theta + y\sin\theta\| = \|x+iy\|_{\mathfrak{T}}.\end{aligned}$$

Therefore, if A is an isometry for $\|\cdot\|$, then $e^{i\theta}A$ is an isometry for $\|\cdot\|_{\mathfrak{T}}$. However, in general the Taylor complexification norm allows isometries different from $e^{i\theta}A$ with A a (real) isometry of $\|\cdot\|$. In this talk the relation between the isometries for a given norm $\|\cdot\|$ on $\mathbb{R}^n$ and the isometries for the complexified norm $\|\cdot\|_{\mathfrak{T}}$ on $\mathbb{C}^n$ will be completely described.

The work of Dijana Ilišević has been fully supported by the Croatian Science Foundation under the project IP-2016-06-1046.

Keywords: isometry, complexification, dual norm, extreme point.

Some algebraic properties of truncated singular integral operators

Dong-O Kang

Chungman National University, Republic of Korea

E-mail address: *mutjaykang@hanmail.net*

Abstract

Nice characterizations of self-adjoint, isometric, coisometric and normal truncated singular integral operators with Cauchy kernel on the unit circle. The main tool is to reduce the commutator of a truncated singular integral operators and the shift operator to some finite rank operators associated with the symbol function of the truncated singular integral operator.

Keywords: singular integral operator, truncated Toeplitz operator, Hankel operator.

The Brown-Halmos theorem for a pair of abstract Hardy spaces

Alexei Karlovich

Universidade Nova de Lisboa, Portugal

E-mail address: *oyk@fct.unl.pt*

Abstract

Let $H[X]$ and $H[Y]$ be abstract Hardy spaces built upon Banach function spaces X and Y over the unit circle $\mathbb{T}$. We prove an analogue of the Brown-Halmos theorem for Toeplitz operators T_a acting from $H[X]$ to $H[Y]$ under the only assumption that the space X is separable and the Riesz projection P is bounded on the space Y. In particular, we show that

$$\|a\|_{M(X,Y)} \le \|T_a\|_{\mathcal{B}(H[X],H[Y])} \le \|P\|_{\mathcal{B}(Y)}\|a\|_{M(X,Y)},$$

where $M(X,Y)$ is the space of all pointwise multipliers from X to Y. We specify our results to the case of variable Lebesgue spaces $X = L^{p(\cdot)}$ and $Y = L^{q(\cdot)}$ and to the case of Lorentz spaces $X = Y = L^{p,q}(w)$, $1 < p < \infty$, $1 \le q < \infty$ with Muckenhoupt weights $w \in A_p(\mathbb{T})$.

This is a joint work with Eugene Shargorodsky (King's College London, UK).

Keywords: Toeplitz operator, abstract Hardy space, pointwise multiplier.

On C–symmetric completions

Cristina Câmara[1], Kamila Kliś-Garlicka[2] and Marek Ptak[2,3]

[1] *Center for Mathematical Analysis, Geometry and Dynamical Systems Mathematics Department, Instituto Superior Técnico, Universidade de Lisboa, Portugal*

[2] *University of Agriculture in Krakow*

[3] *Pedagogical University in Krakow*

E-mail address: *rmklis@cyfronet.pl*

Abstract

Suppose that $\mathcal{H}$ is a complex Hilbert space and $\mathcal{H} = \mathcal{H}_1 \oplus \mathcal{H}_2$. Denote by $L(\mathcal{H})$ the algebra of all bounded linear operators on $\mathcal{H}$. Assume that C is a conjugation on $\mathcal{H}$, i.e., $C^2 = id_{\mathcal{H}}$ and $\langle Cf, Cg\rangle = \langle g, f\rangle$ for $f, g \in \mathcal{H}$. An operator $A \in L(\mathcal{H})$ is C–symmetric if $CAC = A^*$.

During the talk we will discuss the following problems: Suppose that $A_1 \colon \mathcal{H} \to \mathcal{H}_1$ is a bounded operator. When there exists a C–symmetric completion of A_1 into $L(\mathcal{H})$? If such completion exists, how to describe all possible completions and characterise their invariants? In particular we will consider completions of asymmetric truncated Toeplitz operators.

Keywords: conjugation, completion, asymmetric truncated Toeplitz operator.

Weighted composition operators commuting with Hermitian weighted composition operators on H^2

Eungil Ko

Departments of Mathematics, Ewha Womans University, Republic of Korea

E-mail address: *eiko@ewha.ac.kr*

Abstract

In this paper we study the weighted composition operators $W_{g,\psi}$ commuting with Hermitian weighted composition operators $W_{f,\varphi}$ on the Hardy space H^2. In particular, we investigate which combinations of symbol functions g and ψ give rise to the weighted composition operators commuting with such a $W_{f,\varphi}$ on H^2. We also show that these $W_{g,\psi}$ are normal.

References

[1] P. Bourdon and S. K. Narayan (2010) *Normal weighted composition operators on the Hardy space $H^2(\mathbb{D})$*, J. Math. Anal. Appl. **367**, 278–286.

[2] B. A. Cload (1997) *Commutants of composition operators*, Ph.D. Thesis, Univ. of Toronto.

[3] C. C. Cowen (1978) *The commutant of an analytic Toeplitz operator*, Trans. Amer. Math. Soc. **239**, 1–31.

[4] C. C. Cowen (1981) *Commuting analytic functions*, Trans. Amer. Math. Soc. **265**, 69–95.

[5] C. C. Cowen (1983) *Composition operators on* H^2, J. Operator Theory **9**, 77–106.

[6] C. C. Cowen (1988) *Linear factional composition operators on* H^2, Int. Eq. Op. Th. **11**, 151–160

[7] C. C. Cowen, S. Jung, and E. Ko (2014) *Normal and cohyponormal weighted composition operators on* H^2, Operator Theory: Advances and Applications, **240**, 69–85.

[8] C. C. Cowen and E. Ko (2010) *Hermitian weighted composition operators on* H^2, Trans. Amer. Math. Soc. **362**, 5771–5801.

[9] C. C. Cowen and B. D. MacCluer (1995) *Composition operators on spaces of analytic Functions*, CRC Press, Boca Raton.

[10] J. A. Deddens (1972) *Analytic Toeplitz and composition operators*, Canad. J. Math. **24**, 859–865.

[11] R. E. Greene and S. G. Krantz (2002) *Function theory of one complex variable*, Grad. Studies **40**, Amer. Math. Soc.

[12] J. K. Han, H. Y. Lee, and W. Y. Lee (1999) *Invertible completions of* 2×2 *upper triangular operator matrices*, Proc. Amer. Math. Soc., **128**, 119–123.

[13] H. Radjavi and P. Rosenthal (1973) *Invariant subspaces*, Springer-Verlag.

[14] A. L. Shields and L. J. Wallen (1974) *The commutants of certain Hilbert space operators*, Indiana Univ. Math. Surveys, Amer. Math. Soc. Providence, **13**, 49–128.

Location of hot spots in thin curved strips

David Krejčiřík[1,2]

[1] *Czech Technical University in Prague*
[2] *Nuclear Physics Institute ASCR in Řež*

E-mail address: *krejcirik@ujf.cas.cz*

Abstract

According to the conjecture of Rauch's from 1974, any eigenfunction corresponding to the principal eigenvalue of the Neumann Laplacian attains its extrema on the boundary of planar domains. After giving an account on the history and validity of the conjecture, we present our own new results for tubular neighbourhoods of curves on surfaces.

This is joint work with Matěj Tušek.

Complex symmetric block Toeplitz operators on the vector-valued Hardy space $H^2_{C^2}$

Ji Eun Lee[1], Dong-O Kang[2], Eungil Ko[3] and Caixing Gu[4]

[1]*Sejong University, Seoul, Republic of Korea*
[2]*Chungnam National University, Daejeon, Republic of Korea*
[3]*Ewha Womans University, Seoul, Republic of Korea*
[4]*California Polytechnic State University, USA*

E-mail address: *jieunlee7@sejong.ac.kr; jieun7@ewhain.net*

Abstract

In this paper, we study complex symmetric block Toeplitz operators on the vector-valued Hardy space $H^2_{C^2}$. In particular, we provide a characterization of complex symmetric block Toeplitz operators with special conjugations. As some applications, we provide examples of such operators.

Keywords: complex symmetry, block Toeplitz operator, Hardy space.

This research was supported by Basic Science Research Program through the National Research Foundation of Korea(NRF) funded by the Ministry of Education, Science and Technology(2016R1A2B4007035).

Examples of conformal welding

Nikolai Makarov

California Institute of Technology, USA

E-mail address: *makarov@caltech.edu*

Abstract

Versions of Minkowski's question mark function are used to mate Julia sets with reflection groups.

The talk is based on the joint work with S.-Y. Lee, M. Lyubich, and S. Mukherjee

Keywords: Schwarz reflection, iterations, mating.

Cesaro means in function spaces

Javad Mashreghi

Laval University, Canada

E-mail address: *javad.mashreghi@mat.ulaval.ca*

Abstract

Taylor polynomials are not the most natural objects in polynomial approximation. However, in most cases Cesaro means help and the resulting sequence of Féjer polynomials are a good remedy. In the context of *local Dirichlet Spaces*, we show that the sequence of Taylor polynomials may (badly) diverge. However, and surprisingly enough, if we properly modify just the last coefficient in the Taylor polynomial, the new sequence becomes convergent. As a byproduct, this also leads to the convergence of Féjer polynomials and de la Vallée Poussin polynomials.

Joint work with T. Ransford.

Keywords: Dirichlet space, approximation.

Factorisation of strictly singular operators on L^p

Martin Mathieu[1,2] **and Pedro Tradacete**[3,4]

[1] *Queen's University Belfast*
[2] *Mathematical Sciences Research Centre*
[3] *Universidad Carlos III de Madrid and ICMAT*
[4] *Departamento de Matematicás*

E-mail address: *m.m@qub.ac.uk; ptradace@math.uc3m.es*

Abstract

We report on work in progress on factorisation of strictly singular operators on L^p spaces with a view towards a characterisation of strictly singular operator multiplication operators.

Keywords: strictly singular operator, factorisation, multiplication operator.

"Arcsine" Law for anharmonic operators

Boris Mityagin

The Ohio State University, USA

E-mail address: *mityagin.1@osu.edu*

Abstract

Consider a Schrödinger operator $A = -\dfrac{d^2}{dx^2} + Q(x)$, where $Q(x) \in C^2(\mathbb{R})$ is a real, even, convex, slowly changing potential. Let $A\psi_k = \lambda_k \psi_k$, $\|\psi_k\| = 1$, $k \in \mathbb{N}$ be a complete system of eigenfunctions, and let the turning points $x_k > 0$ be defined by $Q(x_k) = \lambda_k$. Assume that $Q(x)$ satisfies

$$\lim_{x\to\infty} \frac{Q(tx)}{Q(x)} = t^\beta, \ \beta \geq 2.$$

Rescale measures, or their densities, on $\mathbb{R}$ by

$$\varphi_k(x) = x_k \psi_k^2(x_k x).$$

Then for any f in the Schwartz space on $\mathbb{R}$,

$$\lim_{k\to\infty}\int_{-\infty}^{\infty} f(x)\varphi_k(x)dx = c(\beta)\int_{-1}^{1} f(x)\frac{dx}{(1-|x|^{\beta})^{1/2}}$$

where $c(\beta) = \dfrac{\Gamma(\frac{1}{2}+\frac{1}{\beta})}{2\pi^{1/2}\Gamma(1+\frac{1}{\beta})}$. Such statements, in the context of the theory of orthogonal polynomials, are well known (Rakhmanov, Mhaskar, Saff, Lubinsky).

These are preliminary results of a joint work of the speaker, Petr Siegl (Queen's University, Belfast, UK) and Joseph Viola (University of Nantes, France).

The Corona Theorem in Nevanlinna quotient algebras and interpolating sequences

Artur Nicolau

Universitat Autònoma de Barcelona, Spain

E-mail address: *artur@mat.uab.cat*

Abstract

Let H^∞ be the algebra of bounded analytic functions in the unit disc and let I be an inner function. P. Gorkin, R. Mortini and N. Nikolskii studied the Corona Theorem in the quotient algebra H^∞/IH^∞ and proved that there is no corona if and only if the inner function I satisfies the so called Weak Embedding Property. We discuss an analogous problem for quotients of the Nevanlinna Class and show that in contrast with the previous case, a complete answer can be given in terms of interpolating sequences.

The results are joint work with Xavier Massaneda and Pascal J. Thomas.

Keywords: Corona Theorem, Nevanlinna class, interpolating sequences.

Decomposability in operator algebra modules

Gabriel Matos[1] **and Lina Oliveira**[1,2]

[1] *University of Lisbon*
Instituto Superior Técnico, Portugal
[2] *Centre for Mathematical Analysis, Geometry, and Dynamical Systems*

E-mail address: *gabriel.matos@tecnico.ulisboa.pt; linaoliv@math.tecnico.ulisboa.pt*

Abstract

When is it the case that a finite rank operator in a subspace of bounded linear operators can be split into a sum of finitely many rank one operators in the given subspace?

In all its generality, the question is bound to have a negative answer. However, in appropriate algebraic and topological settings, it may very well happen that each finite rank operator is decomposable. In this talk, we give a brief overview of this problem and present recent results concerning modules of some reflexive Lie operator algebras, stressing the role of their invariant subspace lattices.

Keywords: finite rank operator, Lie algebra, subspace lattice.

Laplace–Carleson embeddings, and consequences for Hankel and Toeplitz operators

Jonathan Partington

University of Leeds, United Kingdom

E-mail address: *j.r.partington@leeds.ac.uk*

Abstract

Beginning with a brief review of the theory of Laplace–Carleson embeddings [2], we show how it gives results on the boundedness of weighted Hankel operators [1].

Next, describing more recent progress, we turn our attention to Laplace–Carleson embeddings in the context of model spaces, discussing work with R. Zawiski [3]; this leads to results on the boundedness of truncated Hankel and Toeplitz operators.

Keywords: Laplace transform, Carleson embeddings, Hankel operators, truncated Toeplitz operators, model spaces.

References

[1] A. Bashar Abusaksaka and J. R. Partington (2017) Diffusive systems and weighted Hankel operators, *Operators and Matrices*, **11**, no. 1, 125–132.

[2] B. Jacob, J. R. Partington and S. Pott (2013) On Laplace–Carleson embedding theorems, *Journal of Functional Analysis*, **264**, 783–814.

[3] J. R. Partington and R. Zawiski (2018) Carleson measures for model spaces, Hankel operators, and controllability of diagonal systems, *in preparation*.

The Group Inverse of the Nivellateur

Pedro Patrício[1,2] **and Robert E. Hartwig**[3,4]

[1]*University of Minho*
[2]*CMAT*
[3]*North Carolina State University*
[4]*Department of Mathematics*

E-mail address: *pedro@math.uminho.pt*

Abstract

The matrix equation $AX - XB = C$ can be written in column form as $G\mathrm{vec}(X) = \mathrm{vec}(C)$, where $\mathrm{vec}(Y) = \begin{bmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_n \end{bmatrix}$ when $Y = \begin{bmatrix} \mathbf{y}_1 & \cdots & \mathbf{y}_n \end{bmatrix}$, and

$$G = I \otimes A - B^T \otimes I$$

is the *nivellateur* of A and B.

Our aim is to find necessary and sufficient conditions for the existence of the group inverse of this matrix in terms of A and B, and to provide expressions for this group inverse.

Keywords: nivellateur, group inverse, matrices over a field.

On the commutants of weighted shifts on directed trees

Piotr Dymek, Artur Płaneta and Marek Ptak

University of Agriculture in Krakow, Poland

E-mail address: *piotr.dymek@urk.edu.pl; artur.planeta@urk.edu.pl; rmptak@cyf-kr.edu.pl*

Abstract

We introduce generalized multipliers for left-invertible analytic operators. We show that they form a Banach algebra and characterize the commutant of such operators in its terms. In the special case, we describe the commutant of balanced weighted shift on directed tree only in terms of its weights.

Keywords: Left-invertible analytic operator, weighted shift on directed tree, generalized multiplier, commutant.

In the footsteps of Pythagoras

William T. Ross

University of Richmond, USA

E-mail address: *wross@richmond.edu*

Abstract

Pythagoras of Samos (c. 570 – 495 BCE) was a mathematician, philosopher, and teacher whose influence is still being felt millennia after he lived. He is known to all school children by this Pythagorean theorem which, in modern notation, gives us a notion of orthogonality and parallelogram laws in Hilbert spaces. In this joint work with Raymond Cheng and Javad Mashreghi, we explore some notions of weak parallelogram laws and how far certain Banach spaces deviate from Hilbert spaces. This talk also explores a more general notion of orthogonality that has applications to problems in complex analysis (zero sets of analytic functions) and prediction theory (ARMA).

Keywords: parallelogram laws, Hilbert spaces, complex analysis, zero sets, prediction theory.

Remarks on Crouzeix's conjecture

Felix Schwenninger[1] **and Thomas Ransford**[2]

[1] *University of Hamburg*
[2] *Université Laval*

E-mail address: *felix.schwenninger@uni-hamburg.de*

Abstract

Crouzeix's conjecture, formulated in 2007, asserts that the numerical range $W(A)$ of a bounded operator A on a Hilbert space is a 2-spectral set, which in particular would imply that the constant C in the statement

$$\exists C > 0 \, \forall p \in \mathbb{C}[z], n \in \mathbb{N}, A \in \mathbb{C}^{n\times n} : \quad \|p(A)\| \leq C \sup_{z\in W(A)} |p(z)|$$

can be chosen to be $C = 2$. Until now, by recent findings due to Crouzeix–Palencia, the best known bound on the constant is $C \leq 1 + \sqrt{2}$. In this talk a variant of the proof of the latter result is presented, which enables us to abstract the involved arguments and reveal their "optimality".

Keywords: Crouzeix's conjecture, functional calculus.

Essential spectrum of block operator matrices due to singularity

Petr Siegl[1] **and Orif Ibrogimov**[2]

[1] *Queen's University Belfast, UK*
[2] *University College London, UK*

E-mail address: *petr.siegl@math.unibe.ch*

Abstract

We analyze the essential spectrum of block operator matrices with possibly non-symmetric (partial) differential entries. In particular, we describe mechanisms of appearance of essential spectra due to singularities in the coefficients. We present results both on abstract operator theoretic level as well as for more particular problems with differential operators entries arising

in physical applications. The proofs rely on Fredholm theory, Schur complements and quadratic form techniques.

Keywords: unbounded block operator matrix, essential spectrum, differential operators with singular coefficients.

Mixed BMO and mixed Hankels

Elizabeth Strouse

Université Bordeaux 1, France

E-mail address: *Elizabeth.Strouse@math.u-bordeaux.fr*

Abstract

I will be speaking about our progress on understanding the historical relationship between Hankel operators and BMO.

Based on work with Stefanie Petermichl.

Keywords: Hankel, Toeplitz, Bounded mean oscillation.

Limit procedures related to reproducing kernel Hilbert spaces

Franciszek Hugon Szafraniec

Instytut Matematyki
Uniwersytet Jagielloński
Kraków, Poland

E-mail address: *umszafra@cyf-kr.edu.pl*

Abstract

Unlike Hamlet's my paraphrase "to truncate or not to truncate" is not a question; it is an excuse to explain the matters the title refers to. Orthogonal polynomials, especially Hermite ones, as those exposed in my last year's talk, are going to guide us in this environment leading finally to the happy conclusion like that in

K. Górska, A. Horzela and F.H. Szafraniec, Coherence, squeezing and entanglement – an example of peaceful coexistence, Chapter 5 in *Coherent States and their applications: A contemporary panorama*, eds. J-P. Antoine, F. Bagarello and J. P. Gazeau, "Springer Proceedings in Physics", Springer, Heidelberg, 2018.

Keywords: RKHS, reproducing kernel Hilbest space test, Zaremba's construction, inverse a direct limits, orthogonal polynomials, Christoffel-Darboux formula, Hermite polynomials, coherent, squeezed and entangled states.

On the relation between the Talbot effect and the flow patterns associated with noncircular jets

Luís Vega

Universidad del Pais Vasco,
BCAM, Spain

E-mail address: *lvega@bcamath.org*

Abstract In this talk I shall present a conjecture about the need to use the Talbot effect in optics to understand some turbulent phenomena, as the flow patterns associated with noncircular jets generated by nozzles with corners. These patterns will be modeled by the so called Localized Induction Approximation (LIA) of the evolution of vortex filaments. More concretely, I will show that LIA is a nonlinear geometric flow which is amenable to having a nonlinear Talbot effect that, besides the usual properties of randomness, multifractality, and intermittency, has also transfer of energy.

Keywords: Noncircular jets, Talbot effect, Binormal curvature flow.

Compactness of Toeplitz operators on Bergman spaces

Jani Virtanen

University of Reading, UK

E-mail address: *j.a.virtanen@reading.ac.uk*

Abstract

Unlike in Hardy spaces, there are nontrivial Toeplitz operators that are compact on Bergman spaces (even with unbounded symbols), but, despite considerable effort, there is still no full characterization of their compactness (or boundedness). For bounded symbols, it is well known that characterizations can be obtained via the Berezin transform, and, alternatively, using Stroethoff and Zhengs condition (Trans. Amer. Math. Soc. 1992) on the norm of the projection of the symbol composed with a Möbius transformation. I focus on the latter and present a recent joint result with Jari Taskinen, which generalized the condition to a large class of unbounded symbols. Some applications to Fredholm theory of Toeplitz operators are also discussed.

Index of authors